An exegesis of transcendental syntax

PhD defense

LIPN – Université Sorbonne Paris Nord

Boris Eng

Logic with proof systems

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Proof systems with inference rules and sequents

$$\text{hypotheses} \vdash \text{conclusion} \qquad \frac{\Gamma, A \vdash B}{\Gamma \vdash A \Rightarrow B} \qquad \frac{\Gamma \vdash A \Rightarrow B \qquad \Gamma \vdash A}{\Gamma \vdash B}$$

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Functions/programs : $(f : x \mapsto a)$, Computation : $t_1 \rightsquigarrow t_2 \rightsquigarrow \ldots \rightsquigarrow t_n$

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Typing systems with typing rules :

context $\vdash$ program : type

$$\frac{\Gamma, x : A \vdash b : B}{\Gamma \vdash (f : x \mapsto b) : A \to B} \qquad \frac{\Gamma \vdash f : A \to B \qquad \Gamma \vdash a : A}{\Gamma \vdash f(a) : B}$$

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Curry-Howard-Lambek correspondence (CHL)

Formal correspondence between logic and computation.

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Logic	**Computation**
Inference rules	Typing rules
Formula	Type
Proof	(Functional) Program
Implication $\Rightarrow$	Function type $\to$
Cut-elimination	Execution/evaluation

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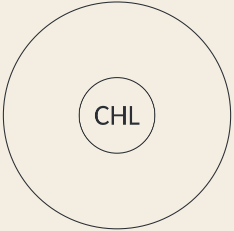
Leads to : cultural mix in proof/type theory, proof assistants, ...

Curry-Howard-Lambek correspondence (CHL)

The unclear status of logic and computation

Programming = Proving. We only discovered a small part.

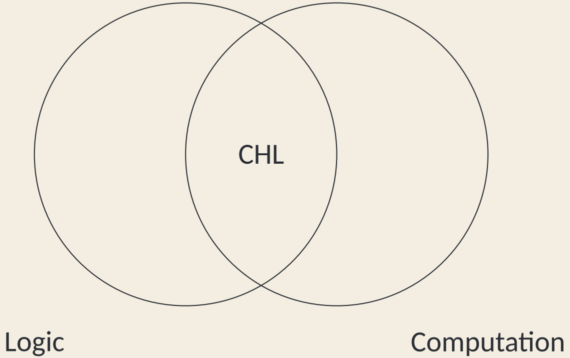


A logico-computational world

Curry-Howard-Lambek correspondence (CHL)

The unclear status of logic and computation

CHL is an intersection. Only some models of computation are logical.



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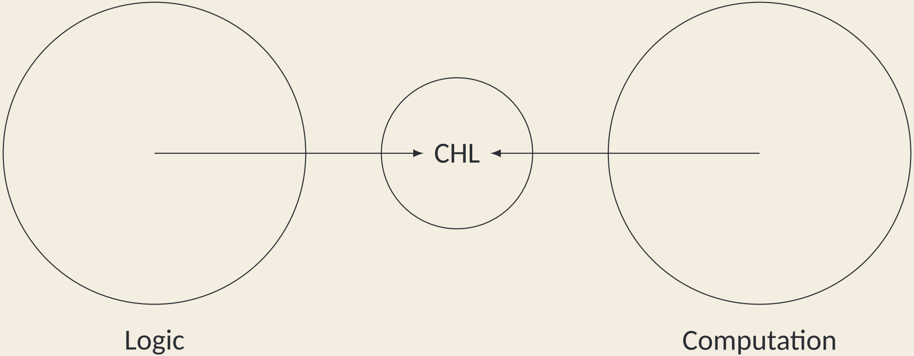
Disjointness. CHL is an identity.



The approach of Girard's transcendental syntax (according to me) ☐

A long trip

Logic

Computation

Curry-Howard-Lambek (CHL)

Proof-structures / Proof-nets

Jean-Yves Girard's works

Transcendental Syntax

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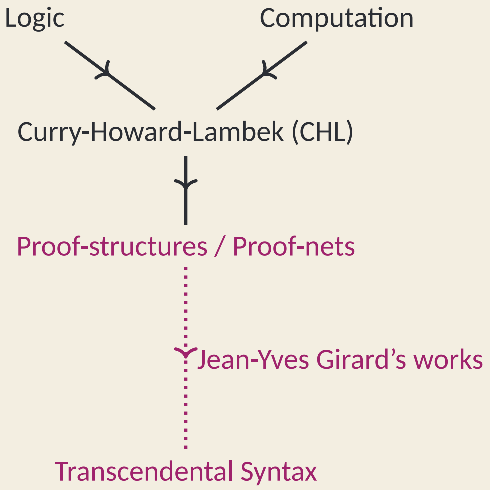

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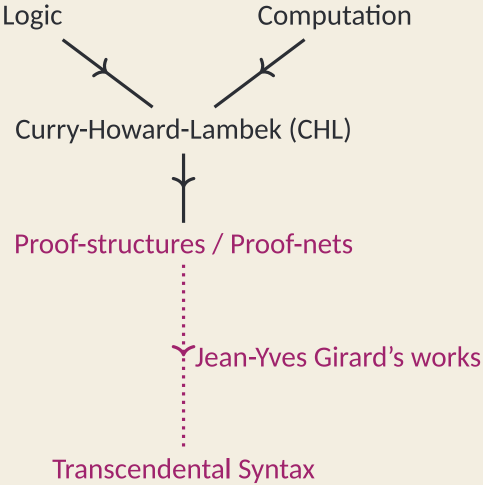

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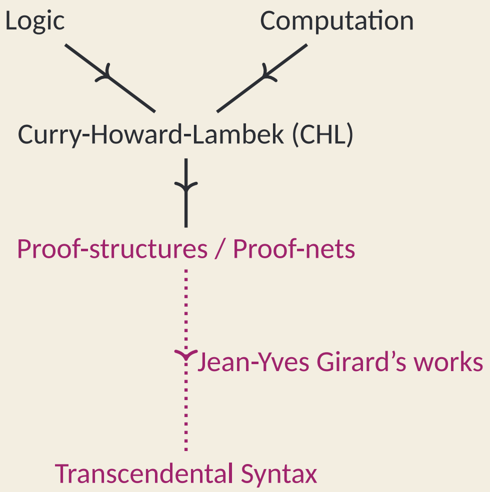

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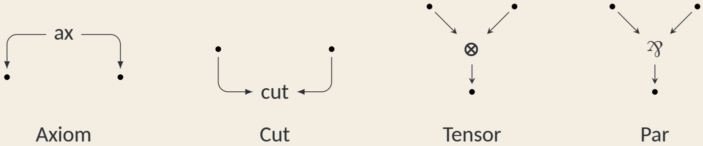
A modern presentation of proofs in multiplicative linear logic

Proof-structures as “aspiring proofs”. A “parallel” presentation of proofs.

↳ case of multiplicative linear logic (MLL)



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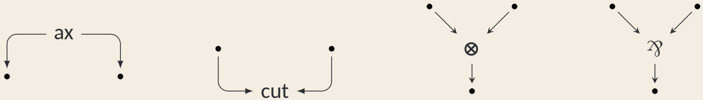
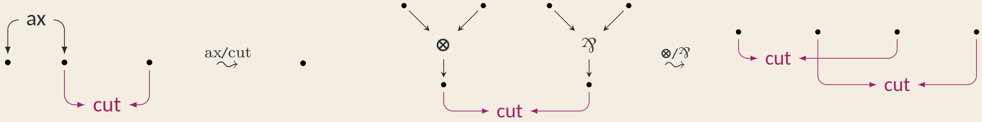


Axiom Cut Tensor Par

They are reducible networks. Cut-elimination ≃ program evaluation/execution :



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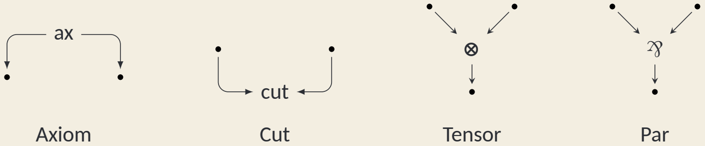

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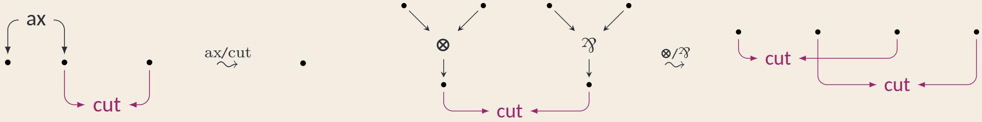

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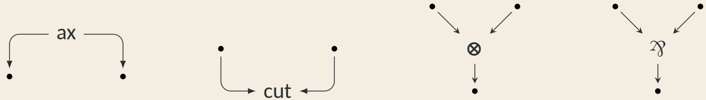

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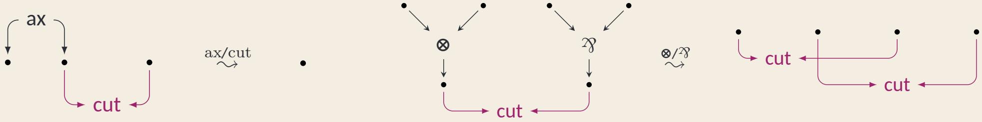

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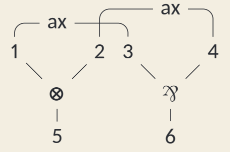
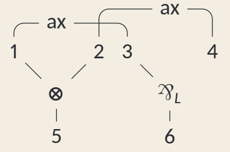
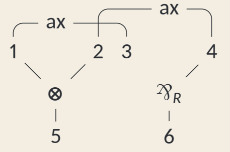
Danos-Regnier correctness criterion by testing

Proof-structure	Switching 1	Switching 2
		

Given a proof-structure :

- it passes all the tests $\longrightarrow$ it is logical (a correct proof = a proof-net)
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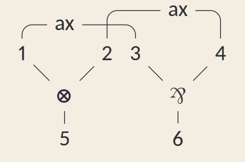	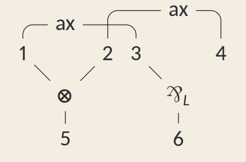	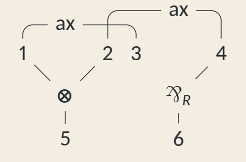

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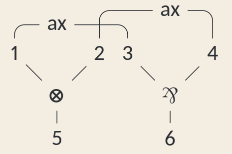
<td>
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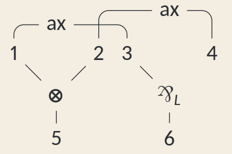
<td>
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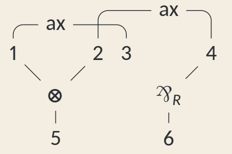
<td>
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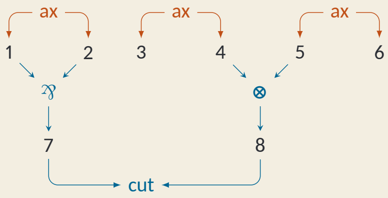
Geometry of Interaction (GoI) : an abstraction of proofs



- Logical rules define constraints on possible paths

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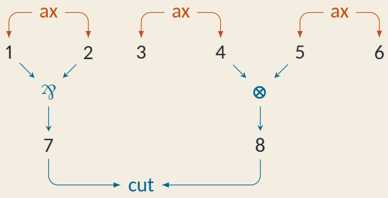

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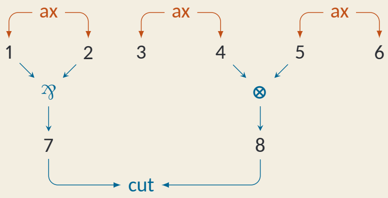

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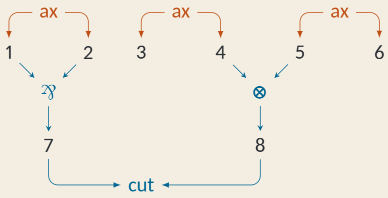

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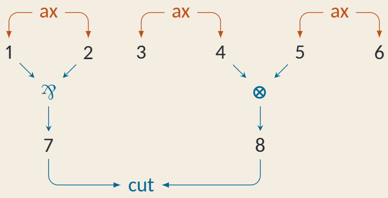

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Transcendental Syntax (TS) : a synthesis

Logic reconstructed from computation

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$$\begin{tikzcd}
\textbf{Computation} & & \textbf{Logic} \\
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& {} \arrow[ur] \arrow[dr] & \\
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- Rays : $r ::= X \mid f(r_1, \ldots, r_k) \mid +f(r_1, \ldots, r_k) \mid -f(r_1, \ldots, r_k)$ e.g. $+f(X)$

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- Constellation : indexed family of stars $\Phi := \phi_1 + \ldots + \phi_n$
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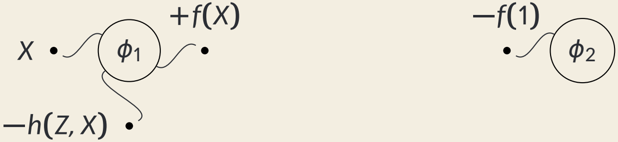

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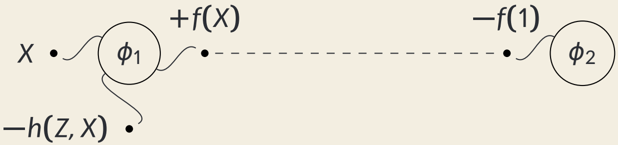
- Constellation : indexed family of stars $\Phi := \phi_1 + \dots + \phi_n$
  ↳ e.g. $[X, +f(X), -h(Z, X)] + [-f(1)]$



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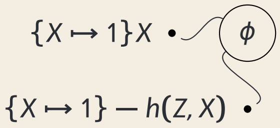

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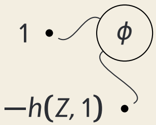

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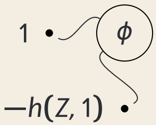

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↳ Variant of Robinson's resolution used in logic programming

Programming with stellar resolution

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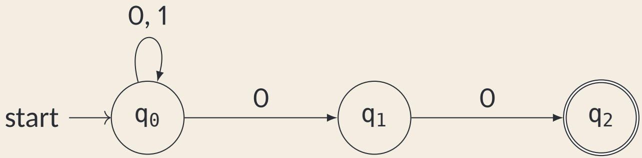

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The problem $[\mathtt{accept}] \overset{?}{\in} \mathrm{Ex}(A^\star)$ simulates word acceptance.

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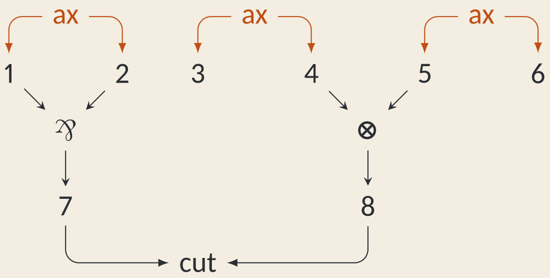
Interpretation of multiplicative linear logic



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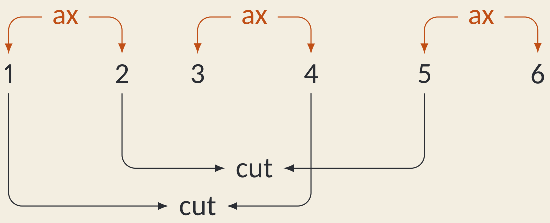

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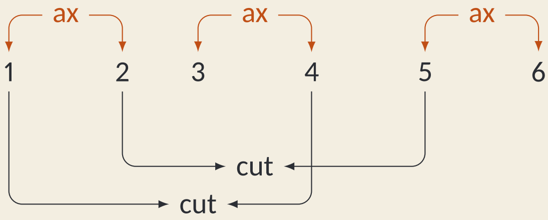

$$[+7(\mathtt{l}\cdot X), +7(\mathtt{r}\cdot X)] + [3(X), +8(\mathtt{l}\cdot X)] + [+8(\mathtt{r}\cdot X), 6(X)]$$

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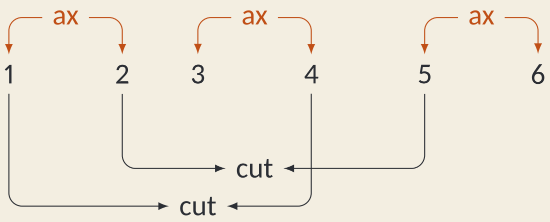

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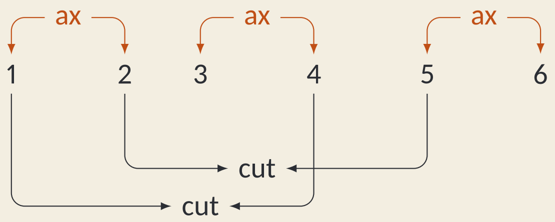

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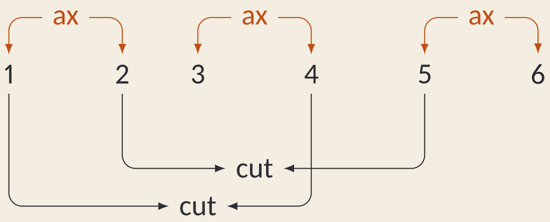

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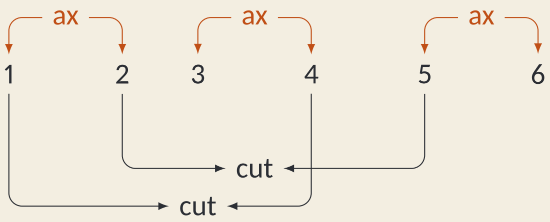

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More general framework : non-proof-structures can enjoy a logical Interpretation

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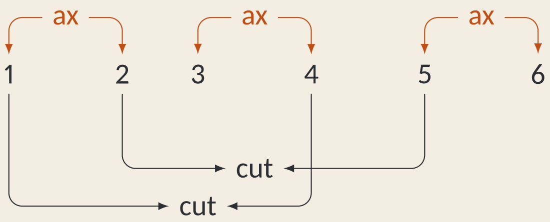

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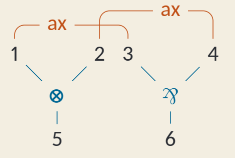
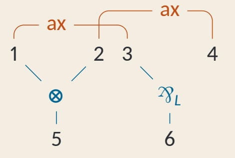
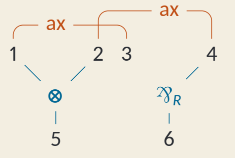
Logical interpretation : Girard's Usine

Proof-structure	Switching 1	Switching 2
		

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<table>
<tr><th>Proof-structure</th><th>Switching 1</th><th>Switching 2</th></tr>
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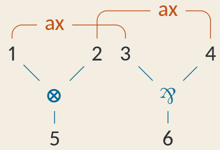
<td>
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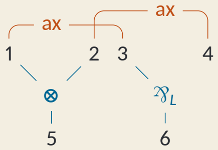
<td>
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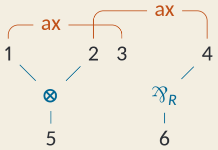
<td>
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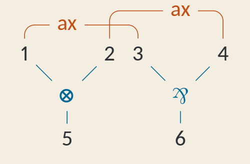	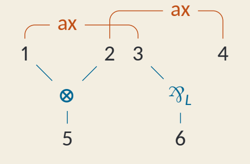	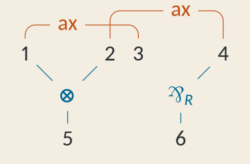

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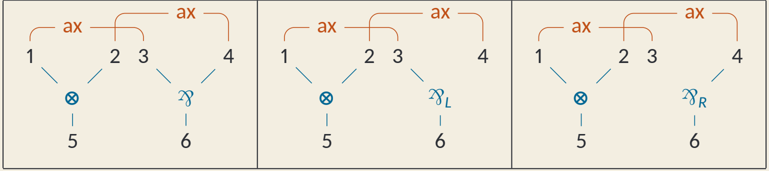

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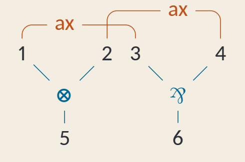	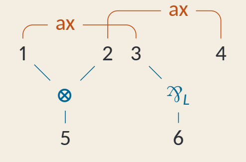	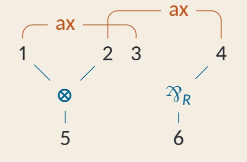

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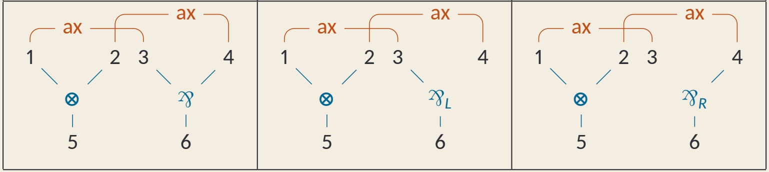

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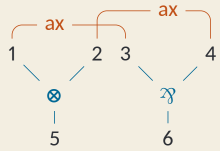	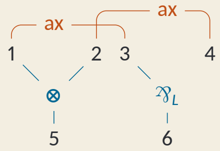	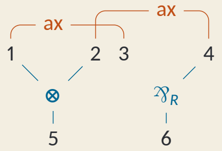

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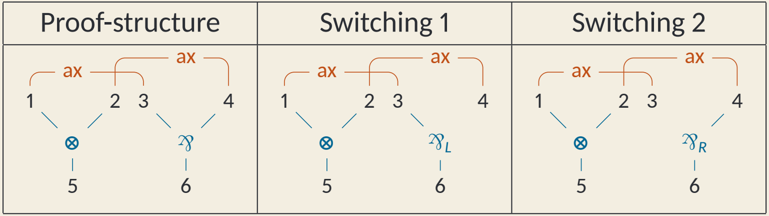

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Appendix

Stellar resolution : execution

Abstract execution

Actual connexion $\longmapsto$ Dependency multigraph (showing compatible rays)

$$\phi_1 — \phi_2 — \phi_2' — \phi_2'' — \phi_3 \qquad\qquad \phi_1 — \phi_2 — \phi_3$$

Diagram. Multigraph homomorphism $\delta : G_\delta \to \mathfrak{D}[\Phi]$

↳ with functions δ_v for each vertex v associating rays to incident edges

↳ G_δ required to be non-empty, finite and connected

Diagram evaluation. Edge contraction by fusion (correct diagram if no failure)

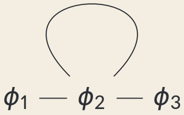
Execution. $\mathrm{Ex}(\Phi) =$ evaluation of all saturated and correct (no failure) diagrams.

Variants. Effective versions with concrete and interactive execution.

Stellar resolution : execution (2/2)

Concrete and interactive execution

Concrete execution. Iterative construction of diagrams / tilings.



$\phi_1 - \phi_2 - \phi_3$ evaluated into ψ_1

$\phi_1 - \phi_2 - \phi_2' - \phi_3$ evaluated into ψ_2

$\phi_1 - \phi_2 - \phi_2' - \phi_2''$

...

Duplicates removed by checking multigraph isomorphism...

Interactive execution. Fusion of stars “on the fly” (without postponing evaluation)